Indian Statistical Institute, Bangalore
B. Math (II)
First semester 2002-2003
Semestral Examination : Statistics (I)
Maximum Score 90

Date : 04-12-2002 **Duration: 3 Hours**

1. Model that is often used for the waiting time X to failure of an item is given by the *pmf* $p_X(k|\theta) = \theta^{(k-1)}(1-\theta); k = 1, 2, ..., ; 0 < \theta < 1$.

   Suppose that we only record the time of failure, if the failure occurs on or before r and otherwise just note that the item has lived at least $r+1$ periods. Let Y denote this censored waiting time. Write down the *pmf* of Y. If $Y_1, Y_2, ..., Y_n$ is a random sample from this censored waiting time distribution, obtain an *mle* of θ. Does your *mle* of θ agree with $\frac{T-n}{T-M}$, where $T = \sum_{i=1}^{n} Y_i$ and $M =$ no. of indices i such that $Y_i = r+1$. [12]

2. Let $X_1, X_2, ..., X_n$ be a random sample from the uniform distribution on the interval (θ_1, θ_2), where both θ_1 and θ_2 are unknown, $-\infty < \theta_1 < \theta_2 < \infty$. Find the *mle's* of θ_1 and θ_2. [10]

3. Let $X_1, X_2, ..., X_n$ be a random sample from the *Poisson* (λ) distribution with parameter $\lambda > 0$. Find sufficient statistics for λ. Give an unbiased estimator for λ. Check whether it attains *CRLB*. If yes, is it *UMVUE* for λ? If not, obtain *UMVUE* for λ and check whether it attains *CRLB*. Give an unbiased estimator for $\psi(\lambda) = e^{-\lambda}$. *Rao-Blackwellize* your estimator and check whether it is *UMVUE*. Does it attain *CRLB* given by $\frac{\left(\frac{d\psi(\lambda)}{d\lambda}\right)^2}{nI(\lambda)}$?

   [20]

4. If X has *Student's t-distribution* with n degrees of freedom, find and identify the distribution of $Y = X^2$.

   [8]

5. If X_1 and X_2 are distributed independently as *exponential* $(\lambda = 1)$, find and identify the distribution of $Y = \frac{X_1}{X_2}$.

   [10]

6. The manufacturer of a certain type of automobile claims that under typical urban driving conditions the automobile will travel at least 20 *km* per litre of petrol. The owner of an automobile of this type notes the mileages that she obtained in her own urban driving conditions when she fills the tank with petrol on 9 different occasions. She finds that the results *km per litre*, on different occasions were as follows :

   15.6, 18.6, 18.3, 20.1, 21.5, 18.4, 19.1, 20.4, 19.0.

   Test the manufacturer's claim by carrying out a test at 5% level of significance. Find the *p-value*. Find 10% *confidence interval* for the expected distance travelled per litre of petrol. List carefully the assumptions you must make. [15]

7. A gambler has been accused of using a loaded die, but he pleads innocent. A record has been kept of last 120 throws. There is a disagreement about how to interpret the data and your services have been sought to decide whether the gambler is innocent. Suppose N_i = number of times i occurs; $1 \le i \le 6$, $\sum_{i=1}^{6} N_i = 120$, and that in the random sample of 120 throws the observed numbers are as follows:

   $N_1 = 8,\ N_2 = 12,\ N_3 = 34,\ N_4 = 32,\ N_5 = 16,\ N_6 = 18.$

   Set up a test at 5% level of significance. Find the *p-value*. List carefully the assumptions you must make.

   [15]